The Richardson Method for Factoring Trinomials

Invented and Developed by Thomas Richardson

The Richardson Method of Factoring Trinomials

Using the example problem: $-176x^2 + 24x - 30x^3$

Step 1: Factor out any common monomial:

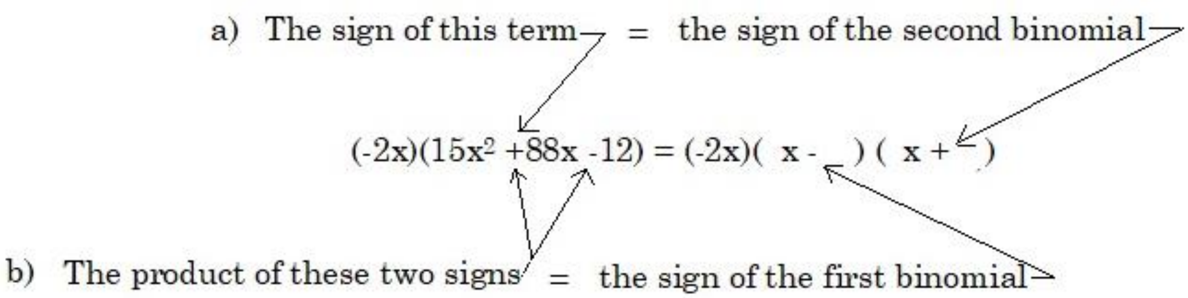
$$-176x^2 + 24x - 30x^3 = (2x)(-88x + 12 - 15x^2)$$

Step 2: Arrange the trinomial in descending powers of the variable:

$$(2x)(-15x^2 - 88x + 12)$$

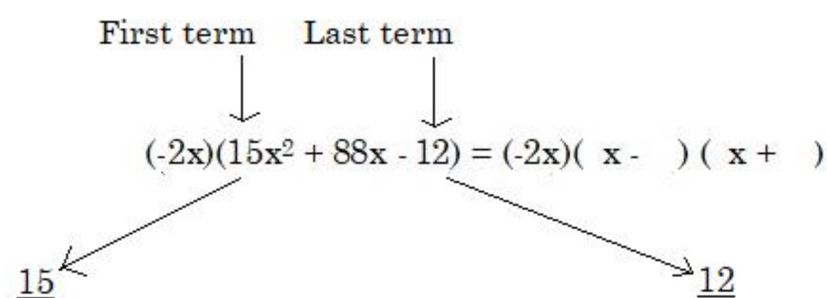
Step 3: If the first term of the trinomial is negative, factor out –1 from the entire expression: $(-2x)(15x^2 + 88x - 12)$

Step 4: Find the signs for the answer's binomials:



[Products of signs: (+)(+) = (+), (–)(–) = (+), (+)(–) = (–)]

Step 5: Find the bifactors of the first and last terms of the trinomial:



$1 \cdot 15$	$1 \cdot 12$
$3 \cdot 5$	$2 \cdot 6$
	$3 \cdot 4$

Bifactors are two numbers that are multiplied together to get the absolute value of a particular product. In the example above, the bifactors 1 and 15 when multiplied together give the product 15, and the bifactors 3 and 5 when multiplied together also give the product 15.

When listing the bifactors, <u>always</u> write the smaller number of the pair first, and list all of the number pairs in exactly the same manner as shown above—with the first numbers going from smallest to largest.

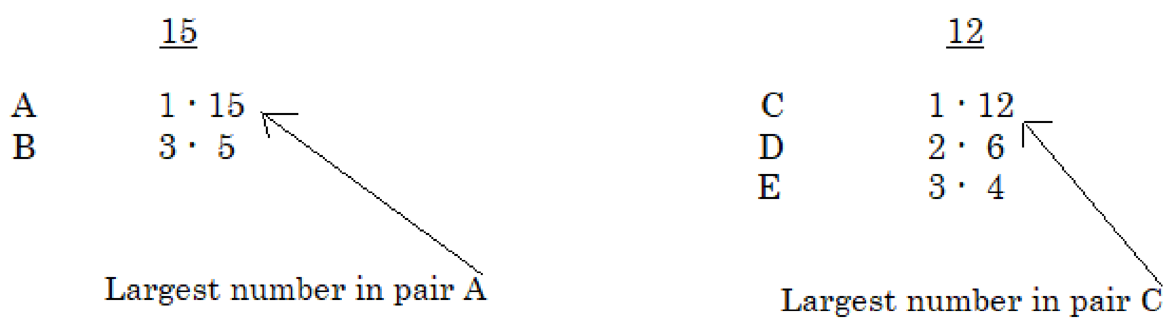
Once you find two bifactors that are repeats (in the example on the previous page, the next bifactors for 15 are 5 · 3, which is a repeat of the previous 3 · 5), you can stop searching for bifactors.

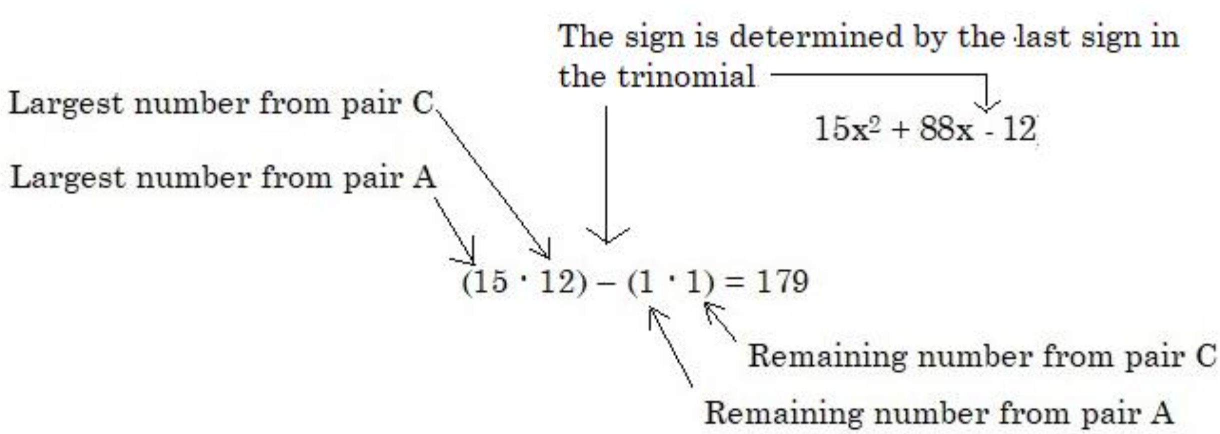
Step 6: Use the bifactors to determine the outer terms of your answer:

$$(-2x)(15x^2 + \underline{88}x - 12) = (-2x)(\ \ x - \ \ )(\ \ x + \ \ )$$

The number "88" is underlined in the equation above because in this step we will be looking for an equation that gives us a result of 88.

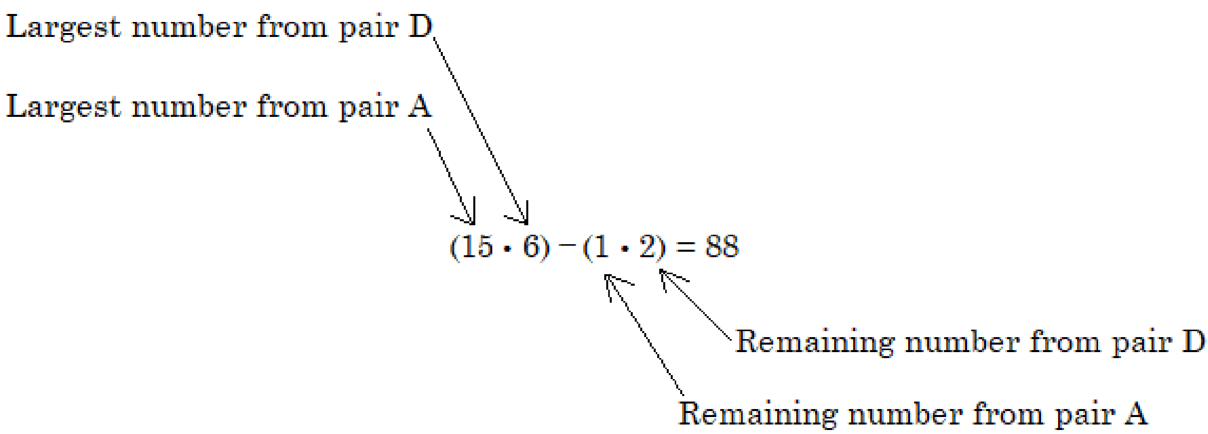
Look at the first number pair from each column of bifactors below (pair A and pair C). Find the largest number from each pair and multiply them together. Then subtract the product of the remaining numbers in the same pairs from it. **We determine if it is an addition or subtraction based on the last sign in the trinomial**. In the trinomial above, the last sign (between 88x and 12) is negative, so we will be subtracting the product of the remaining numbers for this equation. If it had been positive, we would have added the product of the remaining two numbers.



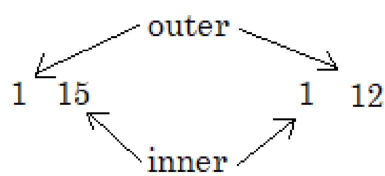


When writing these equations, <u>always</u> write the numbers from the first column before the numbers from the second column (for example, 15 is from the first column and 12 is from the second column, so write $(15 \cdot 12)$ with the 15 before the 12. Writing the numbers in the proper order will become important later in helping you find your answer.

Remember, to solve the example trinomial, we need to find the equation that gives a result of 88. Since the result of this equation is 179, not 88, we will need to keep trying more equations using all the possible pair combinations from the two columns: A&D, A&E, B&C, B&D, and B&E. Pair **combination A&D gives the answer we are looking for and will be the equation we will use to found the outer terms for our trinomial:**



However, if pair A&D or the other pairs mentioned above had not resulted in an equation with the result of 88, we would have continued the search using a different method for choosing the number pairs used in the equation. Instead of starting with the largest number from each pair, we start by looking at the inner and outer terms of each pair. For pairs A&C, it looks like this:



Look at the outer and inner choices from these two pairs and chose the set of numbers which would give the largest product, in this case the inner pair (15 & 1). Multiply them together and subtract the product of the remaining two numbers:[1]

$$(15 \cdot 1) - (1 \cdot 12) = 3$$

Since this equation does not give a result of 88, it is not the equation we are looking for, and we would keep going doing the same thing with the other pair combinations used above: A&D, A&E, B&C, B&D, and B&E. However for our example, we did not have to test all of these pair combinations as we had already found our equation that gives a result of 88:

---

[1] We are still subtracting based on the last sign of the trinomial and still writing the column 1 numbers before the column 2 numbers.

$$(\underline{15} \cdot \underline{6}) - (1 \cdot 2) = 88$$

The first two numbers in this equation (underlined above) give us the outer terms of our binomial answers, and the remaining two numbers give us the inner terms. Put the first number (15) in the first outer term spot and the second number (6) in the second outer term spot of your answer.

$$(-2x)(15x^2 + 88x - 12) = (-2x)(15x - \quad)(\quad x + 6)$$

Outer terms

Step 7: Finding the inner terms of the trinomial

It is now time to finish factoring our trinomial by finding the inner terms of the binomials. In Step 6, we found the following equation:

$$(\underline{15} \cdot \underline{6}) - (1 \cdot 2) = 88$$

We have placed the first two numbers (15 & 6) into our trinomial and are ready to place the second two numbers (1 & 2). At this point, our answer looks like this:

$$(-2x)(15x^2 + 88x - 12) = (-2x)(15x - \quad)(\quad x + 6)$$

Numbers 12 and 6 in the equation directly above tell us where the 1 and 2 go. What number times 6 equals 12? (2) This means that the 2 will go in the first inner term spot and the 1 will go in the second. Thus, our final equation reads:

$$(-2x)(15x^2 + 88x - 12) = (-2x)(15x - 2)(1x + 6)$$

Or, more simply:

$$(-2x)(15x^2 + 88x - 12) = (-2x)(15x - 2)(x + 6)$$

Our trinomial is now completely factored. On the following pages are some practice problems. Follow each step as you do them, and you will find that as you become familiar with the method, it becomes easier to do. You should eventually be able to speed up the process by doing some of the steps in your head.

Practice Problems
(Answers are on the following pages)

1) $x^2 - 8x + 12$

2) $4x^2 + 9x + 2$

3) $4x^2 - 20x + 9$

4) $16x^2 - 16x - 5$

5) $-10x^2 + 3x + 4$

6) $100R^2 - 90R + 20$

7) $A^2B^2 - 7AB + 12$

8) $66x^2 - 76x + 10$

9) $18x^2 - 3x - 3$

10) $39y^2 + 4y - 3$

11) $R^2 - R - 20$

12) $-65x^2 + 58x + 24$

13) $x^2 - x - 12$

14) $20x^2 + 47x - 13$

15) $x^2 - 12x + 35$

16) $y^2 - 2y - 8$

17) $45x^2 - 6x - 16$

18) $x^2 + 13x + 40$

19) $B^2 - 12B - 45$

20) $36x^2 + 63x - 49$

Answers to Practice Problems

1) $x^2 - 8x + 12 = (x - 6)(x - 2)$

<u>1</u>	<u>12</u>
$1 \cdot 1$	$1 \cdot 12$
	$2 \cdot 6$
	$3 \cdot 4$

$(1 \cdot 2) + (1 \cdot 6) = 8$

2) $4x^2 + 9x + 2 = (4x + 1)(x + 2)$

<u>4</u>	<u>2</u>
$1 \cdot 4$	$1 \cdot 2$
$2 \cdot 2$	

$(4 \cdot 2) + (1 \cdot 1) = 9$

3) $4x^2 - 20x + 9 = (2x - 1)(2x - 9)$

<u>4</u>	<u>9</u>
$1 \cdot 4$	$1 \cdot 9$
$2 \cdot 2$	$3 \cdot 3$

$(2 \cdot 9) + (2 \cdot 1) = 20$

4) $16x^2 - 16x - 5 = (4x + 1)(4x - 5)$

<u>16</u>	<u>5</u>
$1 \cdot 16$	$1 \cdot 5$
$2 \cdot 8$	
$4 \cdot 4$	

$(4 \cdot 5) - (4 \cdot 1) = 16$

5) $-10x^2 + 3x + 4 = (-1)(2x + 1)(5x - 4)$

$-1(10x^2 - 3x - 4)$

<u>10</u>	<u>4</u>
$1 \cdot 10$	$1 \cdot 4$
$2 \cdot 5$	$2 \cdot 2$

$(2 \cdot 4) - (5 \cdot 1) = 3$

6) $100R^2 - 90R + 20 = 10(5R - 2)(2R - 1)$

$10(10R^2 - 9R + 2)$

<u>10</u>	<u>2</u>
$1 \cdot 10$	$1 \cdot 2$
$2 \cdot 5$	

$(5 \cdot 1) + (2 \cdot 2) = 9$

7) $A^2B^2 - 7AB + 12 = (AB - 3)(AB - 4)$

<u>1</u>	<u>12</u>
$1 \cdot 1$	$1 \cdot 12$
	$2 \cdot 6$
	$3 \cdot 4$

$(1 \cdot 4) + (1 \cdot 3) = 7$

8) $66x^2 - 76x + 10 = 2(33x - 5)(x - 1)$

$2(33x^2 - 38x + 5)$

<u>33</u>	<u>5</u>
$1 \cdot 33$	$1 \cdot 5$
$3 \cdot 11$	

$(33 \cdot 1) + (1 \cdot 5) = 38$

9) $\mathbf{18x^2 - 3x - 3 = 3(3x + 1)(2x - 1)}$

$3(6x^2 - x - 1)$

<u>6</u>	<u>1</u>
$1 \cdot 6$	$1 \cdot 1$
$2 \cdot 3$	

$(3 \cdot 1) - (2 \cdot 1) = 1$

10) $\mathbf{39y^2 + 4y - 3 = (13y - 3)(3y + 1)}$

<u>39</u>	<u>3</u>
$1 \cdot 39$	$1 \cdot 3$
$3 \cdot 13$	

$(13 \cdot 1) - (3 \cdot 3) = 4$

11) $\mathbf{R^2 - R - 20 = (R + 4)(R - 5)}$

<u>1</u>	<u>20</u>
$1 \cdot 1$	$1 \cdot 20$
	$2 \cdot 10$
	$4 \cdot 5$

$(1 \cdot 5) - (1 \cdot 4) = 1$

12) $\mathbf{-65x^2 + 58x + 24 = (-1)(13x + 4)(5x - 6)}$

$(-1)(65x^2 - 58x - 24)$

<u>65</u>	<u>24</u>
$1 \cdot 65$	$1 \cdot 24$
$5 \cdot 13$	$2 \cdot 12$
	$3 \cdot 8$
	$4 \cdot 6$

$(13 \cdot 6) - (5 \cdot 4) = 58$

13) $\mathbf{x^2 - x - 12 = (x + 3)(x - 4)}$

<u>1</u>	<u>12</u>
$1 \cdot 1$	$1 \cdot 12$
	$2 \cdot 6$
	$3 \cdot 4$

$(1 \cdot 4) - (1 \cdot 3) = 1$

14) $\mathbf{20x^2 + 47x - 13 = (4x - 1)(5x + 13)}$

<u>20</u>	<u>13</u>
$1 \cdot 20$	$1 \cdot 13$
$2 \cdot 10$	
$4 \cdot 5$	

$(4 \cdot 13) - (5 \cdot 1) = 47$

15) $\mathbf{x^2 - 12x + 35 = (x - 5)(x - 7)}$

<u>1</u>	<u>35</u>
$1 \cdot 1$	$1 \cdot 35$
	$5 \cdot 7$

$(1 \cdot 7) + (1 \cdot 5) = 12$

16) $\mathbf{y^2 - 2y - 8 = (y + 2)(y - 4)}$

<u>1</u>	<u>8</u>
$1 \cdot 1$	$1 \cdot 8$
	$2 \cdot 4$

$(1 \cdot 4) - (1 \cdot 2) = 2$

17) $45x^2 - 6x - 16 = (15x + 8)(3x - 2)$

<u>45</u>	<u>16</u>
$1 \cdot 45$	$1 \cdot 16$
$3 \cdot 15$	$2 \cdot 8$
$5 \cdot 9$	$4 \cdot 4$

$(15 \cdot 2) - (3 \cdot 8) = 6$

18) $x^2 + 13x + 40 = (x + 5)(x + 8)$

<u>1</u>	<u>40</u>
$1 \cdot 1$	$1 \cdot 40$
	$2 \cdot 20$
	$4 \cdot 10$
	$5 \cdot 8$

$(1 \cdot 8) + (1 \cdot 5) = 13$

19) $B^2 - 12B - 45 = (B + 3)(B - 15)$

<u>1</u>	<u>45</u>
$1 \cdot 1$	$1 \cdot 45$
	$3 \cdot 15$
	$5 \cdot 9$

$(1 \cdot 15) - (1 \cdot 3) = 12$

20) $36x^2 + 63x - 49 = (12x - 7)(3x + 7)$

<u>36</u>	<u>49</u>
$1 \cdot 36$	$1 \cdot 49$
$2 \cdot 18$	$7 \cdot 7$
$3 \cdot 12$	
$4 \cdot 9$	
$6 \cdot 6$	

$(12 \cdot 7) - (3 \cdot 7) = 63$

Definitions

Bifactors: Two numbers that are multiplied together to get the absolute value of a particular product ($ab = |D|$); a and b are said to be a bi-factor pair of the integer D.

Binomial: A polynomial with two terms. For example, $x + 2$ is a binomial.

Monomial: A polynomial with one term. For example, $5x^3$ is a monomial.

Polynomial: A mathematical expression that consists of one or more terms. For example, $5x^3 + 8x^2 - 6xy + 3$ is a polynomial with 4 terms.

Trinomial: A polynomial with three terms. For example, $x^2 + 3x + 2$ is a trinomial.